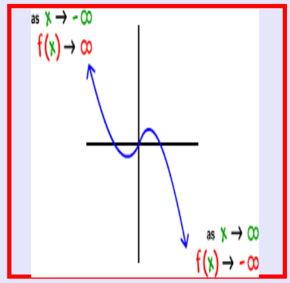
Math 245

Spring 2022

Lecture 41



Given $f(x) = \dfrac{x}{x-4}$

1) Rational function

2) Domain → Deno. $\neq 0$ → $x-4 \neq 0$ → $x \neq 4$
→ $(-\infty,4)\cup(4,\infty)$

3) Vertical Asymptote → $x=4$

4) Y-Int → $x=0$ → $f(0) = \dfrac{0}{0-4} = \dfrac{0}{-4} = 0$
→ $(0,0)$

5) x-Int → $y=0$ → $f(x)=0$ → Num. $=0$ → $x=0$
→ $(0,0)$

6) Horizontal Asymptote

Deg. of numerator = Deg. of denominator

H. A. → $y = \dfrac{\text{Lead. Coef. of Num.}}{\text{Lead. Coef. of Deno.}} = \dfrac{1}{1} = 1$ → $y=1$

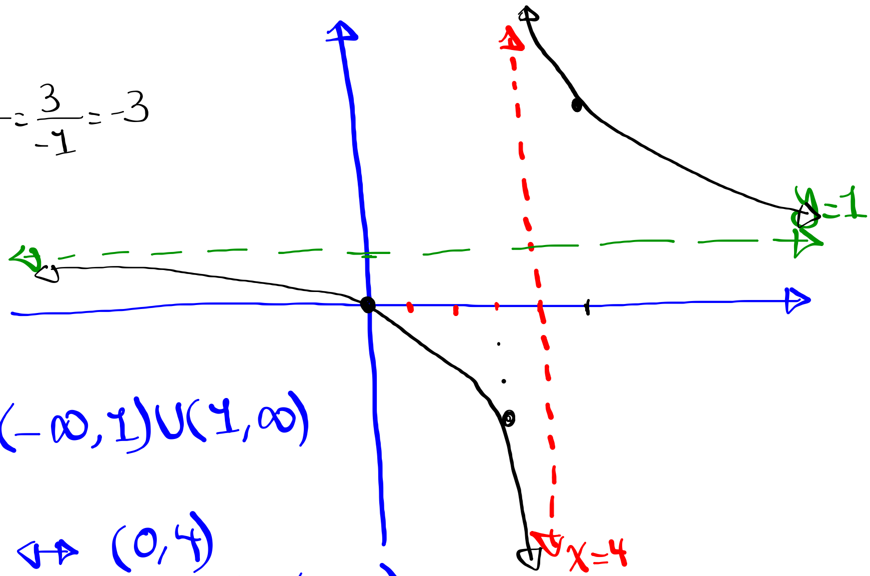
If $x=3$

$y = \frac{3}{3-4} = \frac{3}{-1} = -3$

If $x=5$

$y = \frac{5}{5-4} = \frac{5}{1}$

$= 5$



Range $(-\infty, 1) \cup (1, \infty)$

$f(x) < 0 \Leftrightarrow (0, 4)$

$f(x) > 0 \Leftrightarrow (-\infty, 0) \cup (4, \infty)$

$f(x) = \frac{x}{x^2 - 1}$

1) Rational Function

2) Domain: $x^2 - 1 \neq 0 \rightarrow x^2 \neq 1 \rightarrow x \neq \pm 1$

$\rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

3) V.A. $\rightarrow x = 1$, $x = -1$

4) Y-Int $(0,0)$, x-Int $(0,0)$

5) H.A. Deg. of numerator < Deg. of deno.

$\rightarrow y = 0$

6) $f(-x) = \frac{-x}{(-x)^2 - 1} = \frac{-x}{x^2 - 1} = -\frac{x}{x^2 - 1} = -f(x)$

when $f(-x) = f(x) \rightarrow$ even function $\rightarrow$ Symmetric with respect to Y-axis

when $f(-x) = -f(x) \rightarrow$ odd function

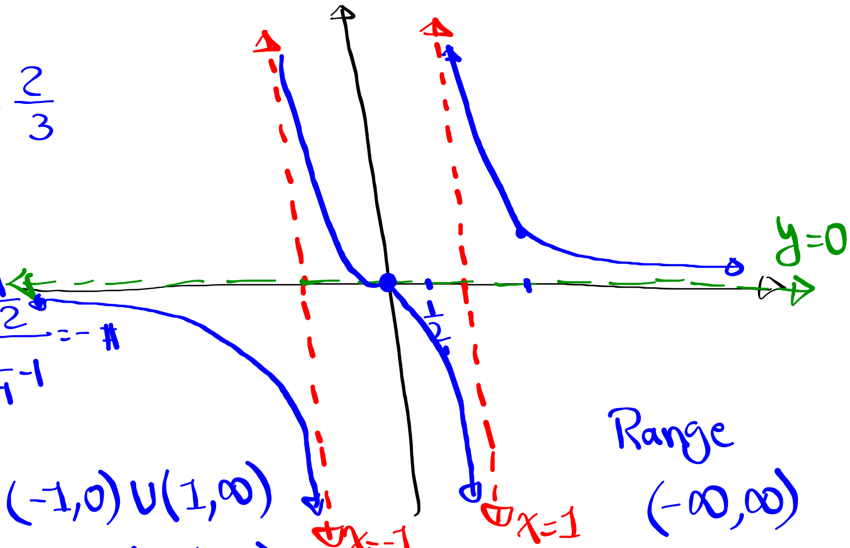
$\rightarrow$ Symmetric with respect to the Origin.

If $x=2$

$$y = \frac{2}{2^2-1} = \frac{2}{3}$$

If $x = \frac{1}{2}$

$$y = \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2 - 1} = \frac{\frac{1}{2}}{\frac{1}{4}-1} = -\#$$



$f(x) > 0 \iff (-1,0) \cup (1,\infty)$

$f(x) < 0 \iff (-\infty,-1) \cup (0,1)$

Range $(-\infty,\infty)$

! Factorial

$n!$ n-Factorial

$n! = n(n-1)(n-2)(n-3)\cdots 3\cdot 2\cdot 1$

$4! = 4\cdot 3\cdot 2\cdot 1 = \boxed{24}$

$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = \boxed{720}$

$\boxed{0! = 1}$ Always

$$\frac{7!}{5!} = \frac{7\cdot 6\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}} = 7\cdot 6 = \boxed{42}$$

Simplify: $$\frac{9!}{4!\cdot 5!} = \frac{\overset{3}{\cancel{9}}\cdot \cancel{8}\cdot 7\cdot 6\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}} = 3\cdot 7\cdot 6 = \boxed{126}$$

Simplify: $$\frac{10!}{7!\cdot 3!} = \frac{10\cdot \overset{3}{\cancel{9}}\cdot \overset{4}{\cancel{8}}\cdot \cancel{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 10\cdot 3\cdot 4 = \boxed{120}$$

Combination Formula

$${}_nC_r = \frac{n!}{r! \cdot (n-r)!}$$

$${}_5C_2 = \frac{5!}{2! \cdot (5-2)!} = \frac{5!}{2! \cdot 3!} = \frac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{2} \cdot 1 \cdot \cancel{3 \cdot 2 \cdot 1}}$$

$$= 5 \cdot 2 = \boxed{10}$$

$${}_7C_3 = \frac{7!}{3! \cdot (7-3)!} = \frac{7!}{3! \cdot 4!} = \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4!}}{\cancel{3 \cdot 2} \cdot 1 \cdot \cancel{4!}}$$

$$= 7 \cdot 5 = \boxed{35}$$

Simplify ${}_8C_6$

$${}_8C_6 = \frac{8!}{6! \cdot (8-6)!} = \frac{8!}{6! \cdot 2!} = \frac{\overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot \cancel{2} \cdot 1} = 4 \cdot 7 = \boxed{28}$$

Binomial Coef:

$$\binom{n}{r} = \frac{n!}{r! \cdot (n-r)!}$$

Find $\binom{7}{5} = \frac{7!}{5! \cdot (7-5)!} = \frac{7!}{5! \cdot 2!} = \frac{7 \cdot \overset{3}{\cancel{6}} \cdot \cancel{5!}}{\cancel{5!} \cdot \cancel{2} \cdot 1} = \boxed{21}$

Find $\binom{10}{5} = \frac{10!}{5! \cdot (10-5)!} = \frac{10!}{5! \cdot 5!} = \frac{\cancel{10} \cdot \overset{3}{\cancel{9}} \cdot \overset{2}{\cancel{8}} \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$

$$= 3 \cdot 2 \cdot 7 \cdot 6$$

$$= 6 \cdot 42$$

$$= \boxed{252} \checkmark$$

You can do all of this using your Scientific calc.

Google it for your calc.

If you are not successful, let me know